Morphological Corner Detection. Application to Camera Calibration

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Abstract

In this paper we study the application of the Affine Morphological Scale Space (AMSS) to corner detection with subpixel precision. Corner detection techniques are in general very sensitive to noise, so some kind of filtering is usually needed to remove noise before estimating the corner location, however, the filtering procedure changes the location of the corner, so the filtering introduce errors in the corner location. If we use the AMSS scale space as filtering, we can solve this problem because following the evolution of the maxima of the curvature across the scales we can recover the original location of the corners with subpixel precision. We apply this technique of corner detection to calibrate a camera system using a calibration object.

Keywords

Computer Vision, Geometric and Morphologic Analysis, Stereo Vision, 3D and Range Data Analysis.

1 Introduction

One of the main concepts of vision theory and image analysis is *multiscale analysis* (or "*scale space*"). Multiscale analysis associates, with an original image $u(0) = u_0$, a sequence of simplified (smoothed) images $u(t, x, y)$ which depend upon an abstract parameter $t > 0$, the *scale*. The image $u(t, x, y)$ is called *analysis of the image* u_0 *at scale* t, (see [1], [9], [13] for more details).

The datum of $u_0(x, y)$ is not absolute in perception theory, but can be considered as the element of an equivalence class. If A is any affine map of the plane, $u_0(x, y)$ and $u_0(A(x, y))$

can be assumed equivalent from a perceptual point of view. Last but not least, the observation of $u_0(x, y)$ does not generally give any reliable information about the number of photons sent by any visible place to the optical sensor. Therefore, the equivalence class in consideration will be $g(u_0(A(x, y))$, where g stands for any (unknown) contrast function depending on the sensor. These considerations lead us to focus on the only multiscale analysis which satisfies these invariance requirements: the Affine Morphological Scale Space (AMSS). This multiscale analysis can be defined by a simple Partial Differential Equation

$$u_t = t^{\frac{1}{3}}(u_y^2 u_{xx} - 2u_x u_y u_{xy} + u_x^2 u_{yy})^{\frac{1}{3}} \tag{1}$$

where $u(t, x, y)$ denotes the image analyzed at scale t and the point (x, y). This Multiscale Analysis has been introduced by Alvarez, Guichard, Lions and Morel in [1].

The remainder of this article is organized as follows: Section 2 presents the technique that we propose to estimate the corner location based on the evolution of the extrema of the curvatures across the scale using the AMSS scale space. In section 3 we present a general overview of the problem of multiple camera system calibration. In section 4 we present an application of the proposed corner detector to camera calibration using a calibration grid. Finally, in section 5, we present the main conclusions of this paper.

2 A robust morphological corner detector

The differential operator of the right part of equation (1) is not new in the framework of corner analysis. For instance, Kitchen and Rosenfeld [8] proposed a measure of cornerness based on the local maxima of the operator

$$\frac{u_y^2 u_{xx} - 2u_x u_y u_{xy} + u_x^2 u_{yy}}{u_x^2 + u_y^2}$$

which corresponds to the second directional derivative in the direction orthogonal to the gradient. The curvature in a point (x, y) of a level line passing by (x, y) is defined by the operator:

$$curv(u) = \frac{u_y^2 u_{xx} - 2u_x u_y u_{xy} + u_x^2 u_{yy}}{(u_x^2 + u_y^2)^{\frac{3}{2}}} \tag{2}$$

The corner detection is based on the threshold of the absolute value of the extrema of this operator.

The main problem with these kind of local measures for corner detection is that they are very sensitive to noise. To avoid this problem, different authors, see for instance [10], [12], and [5], have proposed models based on the gaussian linear multiscale analysis, which corresponds to the convolution of the original image with gaussian functions of increasing width.

The AMSS multiscale analysis presents the advantage that we know, analytically, the displacement of the corner location across the scales. Indeed, in [2], authors show that if (x_0, y_0) is the location of a corner (extremum of the curvature) in the original image $u_0(x, y)$, then if α is the angle of the corner and $\vec{b} = (b_x, b_y)$, is the unit vector in the direction of the bisector line of the corner, then, the location $(x(t), y(t))$ of the extrema of the curvature across the scales is given by the expression:

$$(x(t), y(t)) = (x_0, y_0) + tan(\frac{\alpha}{2})^{-\frac{1}{2}} t(b_x, b_y)$$

of course this result is for an ideal corner without any kind of noise. The technique that we propose in this paper use this result to estimate the corner location in a robust way and can be decomposed in the following steps:

robust morphological corner detector algorithm

1. We compute, using AMSS, the image $u(t_n, x, y)$ at the scale $t_n = t_0 + n\triangle t$, for $n = 1, .., N$, where $\triangle t$ represents the discretization step for the scale and t_0 represents the initial scale that we use to begin to look for corners.

2. We compute for the scale t_0 the location of the extrema of the curvature that we denote by (x_0^i, y_0^i), for $i = 1, .., M$, these points represent the initial candidates to be corners. We follow across the scales the location (x_n^i, y_n^i) of the curvature extrema.

3. For each sequence $(x_n^i, y_n^i)_{n=1,..,N}$, we compute in a robust way (using orthogonal regression and eliminating outliers) the best line which fit the sequence of points, this line corresponds to the bisector line of the corner. We compute also in a robust way the best line passing for the points $(t_n - t_0, d\left((x_n^i, y_n^i), (x_0^i, y_0^i)\right)$. The slope of such line corresponds to the value $tan(\frac{\alpha}{2})^{-\frac{1}{2}}$. After the elimination of outliers the sequence will be reduced to $(x_{n_k}^i, y_{n_k}^i)_{k=1,..,N_k}$. The outlier are usually produced by noise and are located mainly in the first scales. So t_{n_1} represents the first scale where the corner is properly estimated. We also estimate, the direction of the bisector line (b_x^i, b_y^i)

4. Finally, we estimate the location (x^i, y^i) of the associated corner in the original

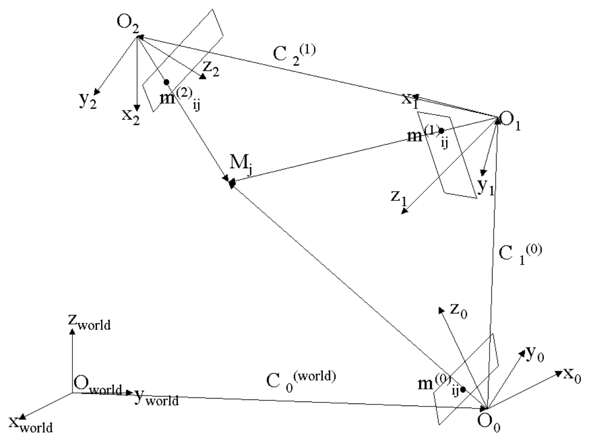

Figure 1: Motion parameters derived from point matches.

image $u_0(x, y)$ as:

$$(x^i, y^i) = (x^i_{n_1}, y^i_{n_1}) - tan(\frac{\alpha}{2})^{-\frac{1}{2}} t_{n_1}(b^i_x, b^i_y)$$

Remarks: For more details on the discretization of the AMSS scale space see [4], for more details on the robust estimation of lines eliminating outliers see the RANSAC technique in [7]

3 Multiple Camera Calibration

The problem of multiple camera calibration consists in recovering the camera positions and orientations with respect to a world coordinate system, using as input data tokens, such as pixels or lines, in correspondence in different images. The figure 1 shows this scenario for a system with two cameras.

The specification of the i-th camera position is the 3D point $C_i^{(world)}$, where the superscript is the reference system in which is the magnitude expressed. The orientation specification is a rotation matrix $R_i^{(world)}$ or any equivalent representation, such as quaternions or Euler angles.

When the images tokens in correspondence are projections of a set of 3D points $\{M_j\}_{j=1..N}$ where N is the number of points, is possible to reconstruct each point 3D expressed in the world coordinate system by simply estimating the intersection point of the line set:

$$\left\{ r_i \equiv C_i^{(world)} + \lambda \overrightarrow{C_i^{(world)} R_i^{(world)} m_{ij}^{(i)}} \right\}_{i=1..N}$$

where $C_i^{(world)}$ are the coordinates of the optical center in the world reference system, and $m_{ij}^{(i)}$ are the coordinates of the projection of M_j in the normalized reference system for the i-th camera. A reference system said to be normalized when the optical center is in the origin and the focal distance is 1. We will assume that the intrinsic parameters of the cameras are known, which allow us to normalize the reference system.

In order to estimate the 3D point intersection of the line set is necessary to know the position of the optical center and the rotation matrix for each one of the cameras. The computation of this parameters resolves the problem of the multiple camera calibration. After estimating these parameters, we can evaluate the accuracy of the solution by projecting the reconstructed 3D points in each camera, and the best solution for the calibration problem is the one that minimizes the energy function:

$$f(C_0^{(world)}, C_1^{(world)}, ..., R_0^{(world)}, R_1^{(world)} ...) = \sum_{i,j} \left\| m'^{(i)}_{ij} - m_{ij}^{(i)} \right\|^2$$

where $m'^{(i)}_{ij}$ is the projection of the reconstructed point M_j in the camera i.

There is no closed-form solution for the minimization of the above energy, and nonlinear minimization methods must be used.

A restriction to take into account in the application of these methods is that the solution must be not only minimum but also valid. A solution is valid when both the matrixes are rotation matrixes and the reconstructed 3D points are always beyond of the image plane, since the reference system in the camera is normalized.

It is important to find a good initial approximation, close to the final solution, to supply as seed input to the nonlinear minimization method to guarantee a fast convergence. This initial solution can be obtain by using the linear methods described in [11]. With this method is possible to calibrate a system with only two cameras. To extend the method to more than two cameras is enough to carry out the calibration for each couple of cameras (the first camera and the second, the second and the third, and so on) and to fit a scale factor for each couple of cameras. This method has the advantage of being linear and the disadvantages of being very noise sensitive (hence the importance of a good estimation of the coordinates points provided by a corner detection technique). On the other hand, the method does not take advantage of having multiples cameras to improve the result of the calibration.

4 Application to Camera Calibration

We have tested the methods previously described using the images in figure 2 as stereo calibration patterns.

Points $m_{ij}^{(i)}$ are the normalized coordinates of the square vertexes in the i-th image.

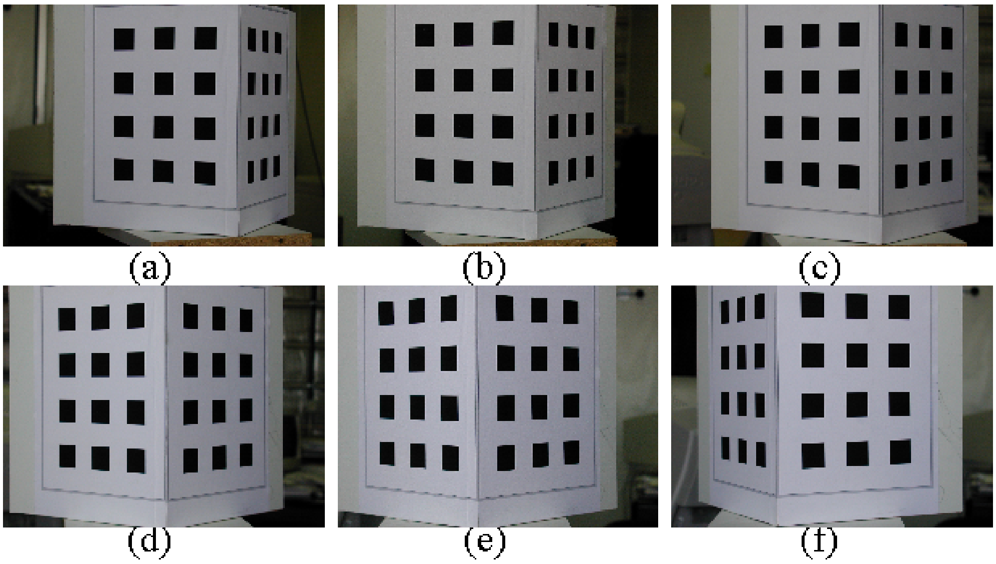

Figure 2: Stereo pattern calibration.

They have been obtained using the morphological corner detector and the correspondence between the points of different images has been carried out manually.

A graphics representation of the result is shown in figure 3. The numerical results have a mean error of 0.1741 of pixels and a standard deviation of 0.0039 of pixels.

These images are the reconstructed solid, shown using a VRML file. The syntax of language allows specifying the position and the orientation of an observer in the world. Next, we used as position and orientation the solution of the multiple camera calibration in order to obtain a similar perspective as in figure 2.

For the basic linear algebra operations, we used the multiprocessor implementation of the BLAS library of the Silicon Graphics. For the problem of finding eigenvalues and eigenvectors and the resolution linear equations system, we used the multiprocessor implementation of the LAPACK of Silicon Graphics. To minimize the error function

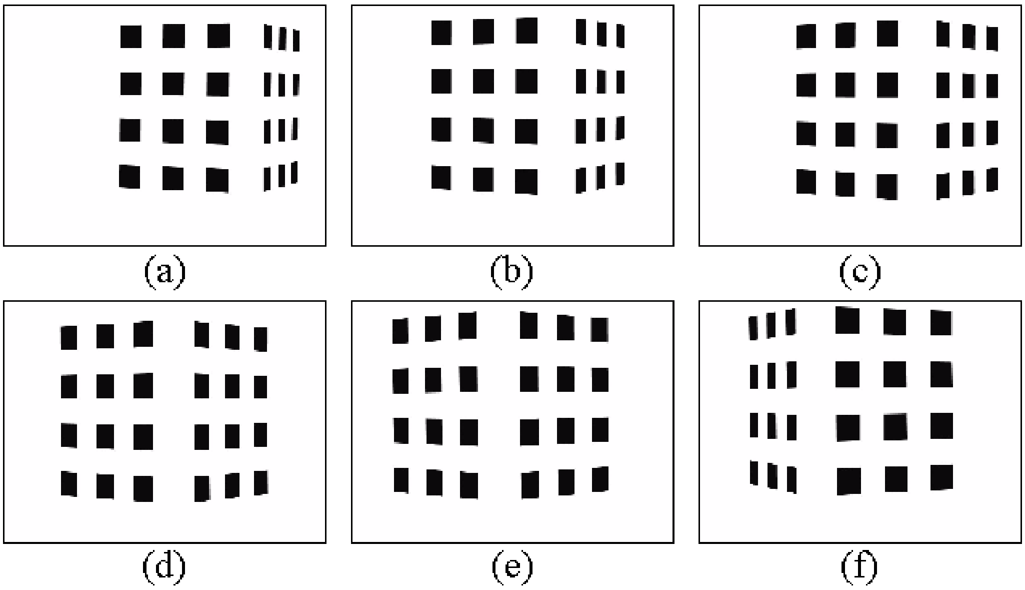

Figure 3: Reconstructed stereo pattern calibration.

we used the implementation of the Levenberg-Marquardt algorithm in the beta gsl 6.0 library of GNU. We parametrized the solution using the three Euler angles, because after each iteration we obtain a rotation matrix. To compute the Jacobian matrix we used the MINPACK library.

5 Conclusions

Corner detection is an important topic in computer vision. The proposed morphological corner detector provides a robust algorithm to estimate corners with subpixel precision. The accuracy of the corner detector algorithm have been tested in the problem of multiple camera calibration. We have computed a very good estimate of the extrinsic calibration parameters based on the proposed morphological corner detector. The mean error between the estimated corner locations and the locations of the corners after reprojection of the

3D points is about 1/10 pixel. This result shows the high accuracy of the proposed morphological corner detector.

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